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Against Deductive Closure

by

PAUL D. THORN

University of Düsseldorf

Abstract: The present article illustrates a conflict between the claim that rational belief sets are closed under deductive consequences, and a very inclusive claim about the factors that are sufficient to determine whether it is rational to believe respective propositions. Inasmuch as it is implausible to hold that the factors listed here are insufficient to determine whether it is rational to believe respective propositions, we have good reason to deny that rational belief sets are closed under deductive consequences.

Keywords: deductive closure, rational belief, full and partial belief

1. Introduction

Within formal theories of rational belief, it is often assumed that rational belief sets are closed under deductive consequences (e.g., within epistemic logic and within the dominant AGM theory of belief revision).[1] The ideal of believing all deductive consequences of the propositions one believes is one that human agents cannot hope to satisfy. For this reason, proponents of formal theories that assume closure generally maintain that the theories properly apply to agents with unlimited deductive abilities, and acknowledge that the prescriptive force of the theories is limited in some respects, in the case of (finite) human beings. In other words, proponents of deductive closure generally adopt the view that deductive closure is an ideal that, nonetheless, exerts some prescriptive force on human beings. This view has some intuitive plausibility, and is supported by common epistemic practice, which appears to presuppose that rational belief tracks logical implications. For example, we routinely assume that rational agents are committed to believe the logical consequences of propositions that they believe, with the consequence that we can compel an agent to repudiate some element of a set of believed propositions by demonstrating that those propositions entail another proposition that the agent is unwilling to accept (Kaplan, 1996, pp. 96–97). We also routinely assume that we can establish the correctness of believing a proposition by showing that it follows from other propositions that we are justified in

1 The canonical sources are Hintikka (1962) and Alchourrón, Gärdenfors and Makinson (1985).

believing (Pollock, 1983, p. 247). Inasmuch as the described epistemic practices ‘seem right’, they provide intuitive support for the claim that (ideal) rational belief sets are closed under deductive consequences. Despite its intuitive plausibility, the claim that rational belief sets are closed under deductive consequences is a subject of ongoing debate.

The present article is motivated by the failure of what was once regarded as a relatively good argument against the claim that rational belief sets are closed under deductive consequences.[2] This (failed) argument derives from the apparent fact that high rational personal probability is a *necessary* condition for rational belief, and from the fact that degree of probability is not generally preserved when we aggregate propositions. In combination, these two facts *suggest* that the conjunction of a set of propositions whose elements are each probable, and objects of rational belief, may be improbable, and therefore not an object of rational belief. So, according to the argument: Given a set of propositions whose elements are each objects of rational belief, it is not the case, in general, that their conjunction will also be an object of rational belief. An advocate of deductive closure can evade the present argument, by maintaining that a personal probability of one is a necessary condition for rational belief, noting that probability one is preserved when we aggregate propositions. But the claim that it is only rational to believe a proposition if its probability is one is extreme, and incorrect as a descriptive claim about the semantics of belief ascriptions.

While the preceding argument against deductive closure may once have been persuasive, its *invalidity* and ultimate failure is now evident, inasmuch as Leitgeb (2013, 2014) has demonstrated the ‘formal possibility’ of relating rational personal probability to rational belief, in such a way that the following two theses are satisfied (where r is a constant, such that $0 < r < 1$):

(DC) *Deductive Closure*: Rational belief sets are closed under deductive consequences.

($LT^{\Rightarrow}$) *The Lockean Thesis, left to right*: Having a rational personal probability of at least r is a *necessary* condition for rational belief (cf. Foley, 1993).

Just what value r has will be of small consequence here, since the problems with (DC) are generated irrespective of r’s value. As such, I will treat r as a numeric constant, and leave the question open as to what value it would have within a true and maximally informative instance of ($LT^{\Rightarrow}$). Given the preceding, my commitment to ($LT^{\Rightarrow}$) may be understood as an existential claim: There exists an s ($0 < s < 1$), such that for all agents, a rational probability of at least s is necessary for rational belief.

2 I articulated the following argument in an unpublished paper (Thorn, ms.) that was presented at several conferences in 2006. A similar argument appears in Foley (2009).

Faced with the failure of the argument described above, I will here endeavour to provide a new and better argument against (DC). In particular, I aim to show (in the context of some other reasonable assumptions, including ($LT^{\Rightarrow}$)) that (DC) conflicts with a very inclusive claim about the factors that are sufficient to determine the rationality of believing respective propositions.

From the outset, it is important to observe that while Leitgeb's theory is the catalyst for the argument presented here, his theory is not the primary or exclusive target of the argument. Inasmuch as Leitgeb's theory maintains (DC) (along with ($LT^{\Rightarrow}$)), the theory lies within the argument's target area. But Leitgeb's theory is rather ambitious, aiming to uphold (DC) along with *both* directions of a 'context dependent' version of the Lockean Thesis. Before considering the connection between Leitgeb's view and ($LT^{\Rightarrow}$), it is useful to compare ($LT^{\Rightarrow}$) with the right-to-left direction of the Lockean Thesis:

($LT^{\Leftarrow}$) *The Lockean Thesis, right to left*: Having a rational personal probability of at least r is a *sufficient* condition for rational belief.

While many find ($LT^{\Leftarrow}$) plausible (including myself), the claim is subject to reasonable doubts. Motivated by Kyburg's Lottery Paradox (Kyburg, 1961) and the desire to uphold the claim that rational belief sets are consistent, Levi (1967) and others rejected ($LT^{\Leftarrow}$), proposing that practical considerations, and/or considerations of expected veritistic value (or epistemic utility), could have a bearing on what rational personal probability threshold was sufficient to license rational belief (cf. Lehrer, 1975).[3] While Levi's rejection of ($LT^{\Leftarrow}$) may have been motivated by the Lottery Paradox, grounds for rejecting ($LT^{\Leftarrow}$) have independent plausibility. If we regard rational personal probability as a function of the strength of an agent's evidence, then it is not implausible to think that the standard of evidence sufficient to license rational belief could vary according to context. Perhaps the evidential standards specific to believing a proposition depend on its subject matter (e.g., mathematical, metaphysical or mundane) and/or the stakes involved in case the belief is in error. Once one allows such contextual variability in evidential standards, it is not implausible to deny that there is any rational personal probability threshold (excluding r = 1) that is sufficient for rational belief. Moreover, if one holds that rational belief sets are consistent, then (by consideration of lottery cases) one will probably want to deny that there is any rational personal probability threshold (excluding r = 1) that is (in all contexts) sufficient for rational belief.

Unlike ($LT^{\Leftarrow}$), ($LT^{\Rightarrow}$) is about as secure as any conceptual claim (outside of logic and mathematics) could be. It is nigh absurd to claim that an agent may

3 Makinson's Preface Paradox (1965) was also important in raising suspicions about deductive closure.

rationally believe some proposition in the case where the agent's rational personal probability for that proposition is less than 0.5 (assuming an epistemic conception of rationality). If one is not convinced that a rational personal probability of 0.5 is necessary for rational belief, then suppose that r is less than 0.5. Notice that the version of ($LT^{\Rightarrow}$) presented here is rather modest, demanding only that $r > 0$.

Up until now, I have assumed a 'context independent' reading of the Lockean Thesis, which has the following consequence: There exists some s ($0 < s < 1$), such that, for all agents, a rational probability of at least s is necessary and sufficient for rational belief. Leitgeb, himself, avoids the Lottery Paradox (understood as a problem for ($LT^{\Leftarrow}$)) by defending a (far weaker) 'context dependent' variant of the Lockean Thesis: For each agent (in the context specific to that agent), there is some value s ($0 < s < 1$), such that a rational probability of at least s is necessary and sufficient for rational belief.[4,5]

Taken individually, each direction of the context dependent Lockean thesis is plausible. The context dependent version of ($LT^{\Rightarrow}$) is no less plausible than the (near indubitable) context independent version of ($LT^{\Rightarrow}$). It is also noteworthy that the context dependent version of ($LT^{\Rightarrow}$) all but implies the context independent version of ($LT^{\Rightarrow}$). If, for each agent, there is a context dependent minimum probability that is necessary for rational belief, then there must be a greatest lower bound on the set of such thresholds. Such a lower bound would then constitute a context independent minimum probability for rational belief. Given the more or less indubitable assumption that this lower bound is greater than zero, the context dependent version of ($LT^{\Rightarrow}$) implies the context independent version. This means that my argument against (DC), by appeal to the context independent version of ($LT^{\Rightarrow}$), could just as well have appealed to the context dependent version of ($LT^{\Rightarrow}$). The preceding considerations also show why Leitgeb's commitment to the context dependent version of ($LT^{\Rightarrow}$), where s takes values greater than 0.5, commits him to the context independent version of ($LT^{\Rightarrow}$), where r is at least 0.5.

The context dependent version of ($LT^{\Leftarrow}$) is more plausible than the context independent version of ($LT^{\Leftarrow}$). All that the former requires is that for each agent, A, there is some s ($s < 1$) such that, for all propositions α, it is rational for A to believe α, if A's rational personal probability for α is at least s. Assuming that rational personal probability one is sufficient for rational belief, the preceding condition is *trivially* satisfied, so long as (for each agent) there is a most probable

4 Such a view is discussed, but not endorsed, by Lin and Kelly (2012).

5 Leitgeb assumes that each agent specific threshold is greater than 0.5. In order to cope with agents whose probabilities are defined over an infinite partition of the space of possible worlds, Leitgeb's theory allows that the threshold for some agents may be one.

proposition (with some probability t) among the propositions that have rational personal probabilities less than one (in which case, a rational personal probability of s, for any s in the interval (t, 1) is sufficient for rational belief).[6] A most probable proposition, with some probability t less than one, is guaranteed to exist, for a respective agent, assuming the agent's rational probabilities are defined over a finite partition of the set of possible worlds. Once we consider agents whose rational personal probabilities are *not* defined over a finite partition, the context dependent version of ($LT^{\Leftarrow}$) is non-trivial. Regardless, by permitting us to vary the value of the threshold, s, the context dependent version of ($LT^{\Leftarrow}$) opens the possibility of avoiding finite versions of the lottery paradox (which is a big advantage, given the puzzles facing existing attempts to formulate infinitary variants of the paradox, cf. Pruss, 2014).

While the context dependent version of ($LT^{\Leftarrow}$) is plausible, the *full* (bi-directional) context dependent version of the Lockean Thesis is subject to reasonable doubt. The full context dependent version of the Lockean Thesis implies the existence of a threshold, for each agent, according to which every proposition above the threshold is an object of rational belief, and every proposition below the threshold is not. Reasonable doubts can be raised here, by appeal to the very same considerations that I mentioned above, as grounds for rejecting the context independent version of ($LT^{\Leftarrow}$): Assuming that the evidential standards specific to believing a respective proposition depend on the subject matter of the proposition and/or the stakes involved in case of error, it is implausible that there is an agent-specific Lockean threshold, for each and every agent. Indeed, since the propositions that an agent considers may concern different subject matters, and the stakes involved in believing respective propositions may vary, it is not unreasonable to maintain that, within respective agents, different standards of belief apply to different propositions.[7]

The content of the preceding paragraphs can be summarized as follows: While ($LT^{\Rightarrow}$) is about as plausible as any conceptual claim could be, both ($LT^{\Leftarrow}$) and the context dependent version of the *full* Lockean Thesis are subject to reasonable doubts. For this reason, the argument presented here is staked upon ($LT^{\Rightarrow}$), and not upon ($LT^{\Leftarrow}$) or upon the context dependent version of the full Lockean

6 On the other hand, the existence of such a t for each agent would not be sufficient to satisfy the context independent version of ($LT^{\Leftarrow}$), which requires the existence of some s ($s < 1$) that applies to all agents, given that for any fixed s ($s < 1$), there exists a t ($t < 1$), such that $t > s$.

7 A weaker (and unobjectionable) context dependent variant of the Lockean Thesis would maintain that for each agent and each proposition (in a given context), there is some s, such that a rational personal probability of at least s is necessary and sufficient for rational belief in the respective proposition. Such a view has some precedent in past discussions of the relationship between degree of belief and full belief (Weatherson, 2005; Ganson, 2008).

Thesis. In this respect, the argument presented here differs from some other contemporary arguments against Leitgeb's theory.[8] That said, Leitgeb's theory represents a prominent attempt to combine ($LT^{\Rightarrow}$) and (DC). So as I proceed, I will be sure to mention where Leitgeb's theory stands with respect to various assumptions that I will make in presenting my argument.

2. Further Preliminaries Concerning (DC) and ($LT^{\Rightarrow}$)

Before proceeding, it is important to make some clarifications regarding (DC) and ($LT^{\Rightarrow}$). As mentioned earlier, (DC) is implausible when considered in application to human agents, due to our limited deductive abilities. In accordance with our cognitive limitations, it is customary to assume that (DC) concerns agents whose deductive abilities are unlimited. In line with this assumption, I regard (DC) as expressing the following claim: For all possible agents, A, with unlimited deductive abilities, the set of propositions that it is rational for A to believe is closed under deductive consequences. In reading the preceding formulation of (DC), I propose that we treat statements about what propositions it is *rational for* an agent to believe as concerning the grounds available to the agent as a basis for forming rational beliefs. In particular, I adopt the convention of saying that it is *rational for* an agent, A, to believe a proposition, α, just in case there are grounds immediately available to A such that if A were to believe that α and base her belief on those grounds, then A's belief that α would be rational. Given the preceding convention, we can recognize cases where an agent's belief in a given proposition is irrational, in virtue of being based upon bad reasons, despite the fact that there are grounds immediately available to the agent that would make the belief rational, if it were based upon those grounds (Kornblith, 1980, pp. 601–602).[9] Next, it should be observed that the literal content of ($LT^{\Rightarrow}$) is quite broad, since one may consistently hold that having a rational personal probability of at least r ($r < 1$) is a necessary condition for rational belief, while also holding that a rational personal probability of one is a necessary condition for rational belief. In order to exclude the preceding possibility, I here propose to treat the application of ($LT^{\Rightarrow}$) in characterizing a theory of rational belief as

8 Peculiarities specific to Leitgeb's theory have been outlined in presentations given by Branden Fitelson and Gerhard Schurz (ms.). Due to a representation theorem presented by Leitgeb (2014, p. 141), those peculiarities would presumably characterize any theory attempting to maintain (DC) along with the context dependent version of the Lockean Thesis (in the context of other plausible assumptions). Further critical work regarding Leitgeb's theory has been presented by David Makinson (ms.) and Hans Rott.

9 The distinction between A's belief that α being rational, and it being rational for A to believe α parallels the distinction between doxastic and propositional justification. For a recent discussion of the latter distinction, see Turri (2010).

appropriate just in case the theory admits cases where the rational personal probability for some proposition is r, and it is rational to believe that proposition.[10]

As a further preliminary to the discussion that follows, observe that the conjunction of (DC) and ($LT^{\Rightarrow}$) is *equivalent* to the claim that for every agent A, there is a proposition α, such that: (a) for all β, it is rational for A to believe that β if and only if α implies β, and (b) A's rational personal probability for α is at least r. It is straightforward to see why it is that the preceding must hold. First note that (DC) is equivalent to the satisfaction of (a) (for some proposition, for each agent). In other words, (DC) holds just in case, for each agent, there is a strongest proposition believed, and all consequences of that proposition are believed. Given (DC), ($LT^{\Rightarrow}$) is equivalent to the satisfaction of (a) and (b) (for some proposition, for each agent). Indeed, since the strongest proposition believed is, itself, believed, ($LT^{\Rightarrow}$) implies (a) and (b), assuming (DC). Conversely, ($LT^{\Rightarrow}$) follows from (a) and (b), given the fact that every logical consequence of a proposition is at least as probable as the proposition itself. Call any α meeting conditions (a) and (b) a "representor" for A, and note, for all agents A: if α is a representor for A, and β is a representor for A, then α is logically equivalent to β.

Leitgeb's theory consists in an elegant way of selecting a representor, for each agent, with the property that every proposition that is not entailed by the representor has probability less than the representor. To achieve this, Leitgeb introduces the notion of P-stability. A proposition, α, is P-stable (where P denotes the relevant probability function) if and only if for all β: if $P(\beta) > 0$ and β is consistent with α, then $P(\alpha|\beta) > 0.5$. Leitgeb then shows that if an agent's representor is P-stable with probability s (and is the logically strongest proposition with probability 1, in the case where $s = 1$), then every proposition that is not entailed by the representor has probability less than s. Since my concern here is to argue against (DC) in the presence of ($LT^{\Rightarrow}$), that is, against the claim that the rational beliefs of agents are fixed by representors, I will not devote specific attention to arguing against Leitgeb's stronger thesis that rational beliefs are fixed by *P-stable* representors.

3. The Determinants of Rational Belief

Despite the formal possibility illustrated by Leitgeb, I argue here that rational personal probability is not related to rational belief, in such a way that ($LT^{\Rightarrow}$) and (DC) are satisfied. Indeed, excluding *inappropriate* applications of ($LT^{\Rightarrow}$), the combination of ($LT^{\Rightarrow}$) and (DC) leads to violations of a very inclusive and highly

10 The application of ($LT^{\Rightarrow}$) in characterizing Leitgeb's theory meets this condition.

plausible claim about the determinants of rational belief. In particular, the combination of ($LT^{\Rightarrow}$) and (DC) leads to cases where the determination of whether a respective agent's belief in a given proposition, α, is rational depends on factors that go beyond: (I) the agent's relevant evidence bearing on α, (II) the type of process that generated the agent's belief that α, (III) the features of the agent's practical situation to which belief in α are relevant, as determined by the agent's attention and interests, and the stakes involved in acting upon the belief that α, (IV) the subject matter of α, (V) the evidential standards applicable to the agent's belief that α, deriving from the relevant features of the social context in which the agent entertains α, and (VI) the agent's degree of doxastic cautiousness, as represented by the minimum rational personal probability that is applicable to the agent as a precondition for rational belief.

It is intended that (I) through (VI) outline a range of facts upon which facts about rational belief supervene. To be more precise, I propose that, for each proposition α, and for any two possible agents who both believe α, a difference in the status of the respective beliefs, as rational or not, implies a difference in at least one of factors (I) through (VI), and (equivalently) no difference in any of factors (I) through (VI) implies that there is no difference in the status of the respective beliefs, as rational or not.[11]

For ease of reference, I will call the preceding supervenience claim the "Relevant Factors Principle". In proposing the Relevant Factors Principle, my intention is to accommodate all those factors that have some *prima facie* plausibility, or some precedent in past discussions of rational belief, regardless of whether I personally think that a particular factor is a potential determinant of rational belief. In other words, I have not striven to propose a minimal characterization of the factors that are determinants of rational belief. Rather I have striven to be ecumenical, proposing something like a disjunction of plausible theories of the determinants of rational belief. Since I am not certain that all of the factors listed by the Relevant Factors Principle are relevant to the determination of rational belief, it is important that the truth of the principle does not rely on the claim that all of the listed factors are relevant. It is straightforward to see that the truth of the principle does not rely on the claim that all of the listed factors are relevant. For example, suppose only (I) and (II) are relevant to determinations of rational belief. Then for each proposition α, and for any two possible agents who both believe α, a difference in the status of the respective beliefs, as rational or not, implies a difference in at least one of (I) or (II). But a difference in at least one of (I) or (II) implies a difference in at least one of (I) through (VI).

11 The sort of supervenience proposed here corresponds to what Kim (1984) calls *strong* supervenience.

As I already mentioned, my intention in proposing the Relevant Factors Principle is to accommodate all those factors that have some *prima facie* plausibility as potential determinants of rational belief. Factor (I) reflects the undisputed claim that relevant evidence has a bearing on rational belief. Factor (II) covers two sorts of consideration. First, it is inclusive of a broadly reliabilist conception of rational belief (cf. Goldman, 1979). Second, it accommodates the idea that whether a belief is rationally held depends not just on the agent's evidence, but on the manner in which the belief is *based* on the evidence (cf. Harman, 1973, p. 26; Pollock, 1986, p. 81). It would, I think, be sensible to maintain that (I) and (II) are exhaustive of the factors that are determinative of the rationality of holding respective beliefs. Indeed, given that our concern is with the norms of what has been variously described as theoretical, epistemic and evidential rationality, it is hard to see why we should think that factors apart from evidence, and the manner in which belief is based upon evidence, could be relevant to determining the rationality of holding a belief (cf. Feldman and Conee, 1985). That said, the form of evidentialism encapsulated by the preceding idea has come under attack in the last fifteen years. Rather than resist anti-evidentialist sentiment, I here accede to intuitions to the effect that pragmatic factors, (III), may be relevant to determining the rationality of belief (cf. Pollock, 1995; Fantl and McGrath, 2002; John Hawthorne, 2004; James Hawthorne, 2009). Having conceded the possible relevance of pragmatic factors, I have admitted a few other factors that appear to be about as plausible: Factor (IV) captures the idea that the epistemic standards applicable to believing a proposition may depend on the proposition's subject matter (Hawthorne, 2009). Factor (V) reflects the idea that different doxastic standards may be applicable to the determination of rational belief, deriving from the social context in which the belief is entertained (cf. DeRose, 1995; Hawthorne, 2004; Hawthorne, 2009). Factor (VI) captures the idea that different agents might rationally adopt different degrees of cautiousness about what they believe, and thereby have different *minimal* standards about the strength of evidence that is required for rational belief (cf. Leitgeb, 2014). Admission of factors (III), (IV), (V) and (VI) (in addition to (I) and (II)) is sensible, since my concern is to propose a principle that is plausible, regardless of the side one takes in the debate over (DC). With this in mind, I have been careful to ensure that the Relevant Factors Principle explicitly mentions all of the factors to which Leitgeb appeals in attempting to motivate his context sensitive version of the Lockean Thesis.[12]

12 Leitgeb also acknowledges that his view depends on the claim that rational belief is partition relative. In section 5, I show that (DC) conflicts with a variant of the Relevant Factors Principle that explicitly allows that an agent's partition of the space of possible worlds is a potential determinant of rational belief.

Beyond the admission of (I) through (VI), notice that a failure to mention *explicitly* some factor that is relevant to determinations of rational belief is not sufficient to falsify the Relevant Factors Principle. For example, it is sometimes claimed that the rationality of a belief is partly dependent upon the explanatory power of the proposition that is the object of the belief, where a proposition's explanatory power is understood as the degree to which it provides a good explanation of the observable data (Schupbach and Sprenger, 2011). Although the Relevant Factors Principle makes no mention of explanatory power, the principle is consistent with the claim that rational belief depends on explanatory power, inasmuch as differences in the explanatory power of a proposition, α, for two agents, invariably imply differences in their evidence bearing on α. More generally, the Relevant Factors Principle is consistent with the claim that some factor, F, is relevant to determinations of rational belief provided the following condition is satisfied: For all propositions, α, and all pairs of agents, A_1 and A_2, who believe α, if A_1 and A_2 differ with respect to F-facts, and in the rationality of their beliefs that α (i.e., one agent's belief is rational and the other's is not), then there is also a difference in at least one of (I) through (VI).[13]

4. Against Deductive Closure

The argument against (DC) proceeds by a reductio ad absurdum. It begins by assuming ($LT^{\Rightarrow}$) and (DC), and employs the following extended example. Consider two agents A_1 and A_2, with unlimited deductive abilities, whose total evidence, and rationally held personal probability functions, P_1 and P_2, exclusively concern two disjoint domains D_1 and D_2, describable by propositional atoms that correspond to orthogonal partitions of the space of possible worlds: $p_1, \ldots, p_n$ for D_1, and $q_1, \ldots, q_m$ for D_2.[14] As a consequence, A_1's evidence has no bearing on truth functional combinations of $q_1, \ldots, q_m$, and A_2's evidence has no bearing on truth functional combinations of $p_1, \ldots, p_m$. Let α_1 be a representor for A_1, and

13 As with explanatory power, it is plausible that the other factors that are typically thought to be relevant to rational theory selection (such as coherence with established background knowledge and ontological simplicity) satisfy the present condition. Since these factors have not been cited within the debate about deductive closure, I will not pause to argue for the preceding claim. Regardless, it is clear that for the relevant pairs of beliefs that are considered in sections 4 and 5 (i.e., A_1's and A_{1*2}'s belief that α_1, and A_2's and A_{1*2}'s belief that α_2), there is no difference in coherence with established background knowledge and no difference in the ontological commitments entailed by the respective beliefs. So we could append the two factors to the Relevant Factors Principle, and use the very same examples to force a conflict with (DC).

14 It may be that there are some propositions that are supported by every body of evidence. If this is so, then the supposed disjointedness in the objects of A_1 and A_2's evidence should be understood as concerning those propositions that could fail to be supported by a body of evidence.

let α_2 be a representor for A_2. (Recall, from section 2, that a representor corresponds to the strongest proposition believed by an agent whose belief set is closed under deductive consequences.) Suppose that $P_1(\alpha_1) = P_2(\alpha_2) = r$, and that A_1's and A_2's degree of epistemic cautiousness is identical, such that the minimum personal probability threshold for rational belief for both A_1 and A_2 is r. Suppose that A_1 and A_2 will proceed within the respective domains D_1 and D_2 via actions that are specific to each domain, where neither domain presents an opportunity to perform a moral or immoral action (or a virtuous or vicious action), and the result of performing various actions are immediate payoffs in units of utility. Assume that both A_1 and A_2 engage in appropriate practical deliberations. Even better, assume that neither A_1 nor A_2 has any practical interests (i.e., A_1 and A_2 are indifferent to what state of affairs obtains). Finally, assume that the contexts in which A_1 and A_2 entertain α_1 and α_2 are thoroughly asocial.

Now observe that it is possible to form a probability function, P_{1*2}, that is defined over truth functional combinations of $p_1, \ldots, p_n$, and $q_1, \ldots, q_m$, that: (1) agrees with P_1 regarding propositions that exclusively concern D_1, (2) agrees with P_2 regarding propositions that exclusively concern D_2, and (3) assigns probabilities to other propositions by treating propositions that exclusively concern D_1 as being probabilistically independent of propositions that exclusively concern D_2. To see how P_{1*2} is formed, notice that the set of possible worlds with respect to D_1 may be identified with conjunctions of the form: $(\neg)p_1\wedge\ldots\wedge(\neg)p_n$ (where "$(\neg)$" indicates that the atom that follows is either negated or unnegated). The set of such conjunctions/worlds may be listed as: $w_{P1}, \ldots, w_{P2^n}$. Similarly, the set of possible worlds with respect to D_2 may be identified with conjunctions of the form: $(\neg)q_1\wedge\ldots\wedge(\neg)q_m$. The set of such conjunctions/worlds may be listed as: $w_{Q1}, \ldots, w_{Q2^m}$. The set of possible worlds with respect to the joint domain of D_1 and D_2 may now be identified with conjunctions of the form: $w_{Pi}\wedge w_{Qj}$ (for respective combinations of w_{Pi} and w_{Qj}). Finally, let $P_{1*2}(w_{Pi}\wedge w_{Qj})$ be identical to $P_1(w_{Pi}) \times P_2(w_{Qj})$, for all such combinations. As an immediate consequence, observe that for any proposition of the form $\phi\wedge\chi$, where ϕ is a truth function of $p_1, \ldots, p_n$, and χ is a truth function of $q_1, \ldots, q_m$, we have $P_{1*2}(\phi\wedge\chi) = P_1(\phi) \times P_2(\chi)$.

It is apparent that adopting the probability function P_{1*2} would be a rational response (though perhaps not uniquely so) for an agent whose total evidence is the aggregate of A_1's and A_2's evidence (but see the last paragraph of this section, where an alternative view is considered). Consider an agent, A_{1*2}, whose total evidence is the aggregate of A_1's and A_2's, whose deductive abilities are unlimited, and who rationally adopts P_{1*2}. Next suppose that A_{1*2}'s degree of doxastic cautiousness is identical to that of A_1 and A_2, so that A_{1*2}'s minimum rational personal probability threshold for rational belief is r. Further, suppose that A_{1*2}

believes exactly the same propositions concerning domain D_1 as A_1, forming them by type identical processes to the ones that produced A_1's beliefs. Similarly, suppose that A_{1*2} believes exactly the same propositions concerning domain D_2 as A_2, forming them by type identical processes to the ones that produced A_2's beliefs. As with A_1 and A_2, assume that the context in which A_{1*2} entertains α_1 and α_2 is thoroughly asocial. At this point, it is not assumed that any of A_{1*2}'s beliefs are rational.

It is already assumed that D_1 and D_2 correspond to orthogonal partitions of the set of possible worlds. In addition, suppose that each of the actions that A_{1*2} is capable of performing has a potential impact on no more than one of D_1 and D_2, and A_{1*2} knows which actions are relevant to which domain. Next, suppose that the actions available to A_{1*2} with respect to D_1 are identical to the ones available to A_1, where A_{1*2}'s payoff for performing respective actions under various D_1 conditions are identical to the payoffs for A_1, and these payoffs are independent of what D_2 conditions obtain (and there is no opportunity for performing moral or immoral actions, etc.). Similarly, suppose that the actions available to A_{1*2} with respect to D_2 are identical to the ones available to A_2, where A_{1*2}'s payoff for performing respective actions under various D_2 conditions are identical to the payoffs for A_2, and these payoffs are independent of what D_1 conditions obtain (and there is no opportunity for performing moral or immoral actions, etc.). As with A_1 and A_2, we may assume that A_{1*2} has no practical interests (i.e., A_{1*2} is indifferent to what state of affairs obtains). Assuming that neither A_1 nor A_{1*2} has any practical interests, neither agent's belief that α_1 is relevant to her practical situation. On the other hand, if we do assume that A_1 and A_{1*2} have practical interests, then the practical bearing of belief in α_1 is the same for both agents. Indeed, while the breadth of A_{1*2}'s practical concerns may exceed those of A_1, the bearing of each agent's belief that α_1 is, by stipulation, limited to D_1, and the interests of the two agents regarding D_1 are, by stipulation, identical. By similar reasoning, the practical bearing of A_2's and A_{1*2}'s belief in α_2 is the same.

Given the suppositions made, note that (i) A_{1*2}'s relevant evidence concerning α_1 is identical to A_1's, (ii) A_{1*2}'s belief that α_1 was formed by the same process type as A_1's, (iii) the features of A_{1*2}'s practical situation to which her belief that α_1 is relevant are identical to the features of A_1's practical situation to which her belief that α_1 is relevant, (iv) the subject matter of A_{1*2}'s belief that α_1 is identical to the subject matter of A_1's belief that α_1, (v) the relevant features of the social context in which A_1 and A_{1*2} entertain α_1 are identical, and (vi) A_{1*2} and A_1 employ the same degree of doxastic cautiousness, as represented by the same minimum rational personal probability threshold for rational belief. So, according to the Relevant Factors Principle, it is rational for A_{1*2} to believe α_1. *Mutatis mutandis*, it is rational for A_{1*2} to believe α_2. By (DC), it is also rational for her to believe $\alpha_1 \wedge \alpha_2$. But $P_{1*2}(\alpha_1 \wedge \alpha_2) = r^2 < r$, since $P_{1*2}(\alpha_1 \wedge \alpha_2) = P_1(\alpha_1) \times P_2(\alpha_2)$. So, by ($LT^{\Rightarrow}$), it is *not* rational for her to believe $\alpha_1 \wedge \alpha_2$. Thus, by reductio, not ($LT^{\Rightarrow}$) or not (DC). But ($LT^{\Rightarrow}$) is clearly the case. So not (DC).

The preceding argument appealed to the claim that P_{1*2} is a rational response to the aggregate of A_1's and A_2's evidence. The crucial consequence of this claim is the claim that $P(\alpha_1 \wedge \alpha_2) = r^2$ is a rational personal probability, given a total body of evidence that is the aggregate of A_1's and A_2's evidence. The only plausible grounds for denying that P_{1*2} is a rational response to the aggregate of A_1's and A_2's evidence, derives from a 'sceptical' view about the attribution of rational personal probabilities. According to such a view, it is rational to adopt personal probabilities in agreement with P_1 and P_2, regarding their respective domains, but it is only rational to form interval-valued personal probabilities that are maximally imprecise with respect to propositions (such as $\alpha_1 \wedge \alpha_2$) that concern both domains, save for what precision can be deduced from the aforementioned agreement with P_1 and P_2. While the present view denies that P_{1*2} is a rational response to the aggregate of A_1's and A_2's evidence, the view does not present an obstacle to the argument presented here. Indeed, the described view maintains that $P(\alpha_1 \wedge \alpha_2) \geq r$ is *not* a rational response to the aggregate of A_1's and A_2's evidence, thereby upholding that it is not rational to believe $\alpha_1 \wedge \alpha_2$, in a situation where it is rational to believe α_1 and α_2.

5. A Variant of the Example

The example of the preceding section features in a reductio ad absurdum of (DC). The reductio is plausible to the extent that the Relevant Factors Principle is plausible. If we extend the Relevant Factors Principle by including additional factors, we can increase the plausibility of the principle. I now describe a variant of the example of the preceding section that applies when the Relevant Factors Principle is extended, and explicitly states that whether an agent's belief in a proposition is rational may depend on: (VII) the *partition* of the space of possible worlds over which the agent's personal probabilities are defined. This extension of the principle is relevant to the debate over deductive closure, since Leitgeb (2014), along with Lin and Kelly (2012), have claimed that rational belief is partition relative, in the course of defending (DC). The example presented here shows that we can grant this claim, and still create problems for (DC).

Within the example of section 4, A_{1*2}'s and A_1's personal probabilities are defined over different partitions of the set of possible worlds (and similarly for A_2). We can modify the example so that their personal probabilities are defined over the same partition (i.e., the joint domain of D_1 and D_2), given the reasonable assumption that it is *permissible* to suspend belief regarding contingent propositions about which one has *no* evidence. In particular, suppose that all features of the example are the same, save that (1) both A_1's and A_2's rational personal probabilities are defined over the joint domain of D_1 and D_2, and (2) A_1 and A_2 suspend belief regarding contingent propositions for which they have no evidence. Given the prior supposition that their respective bodies of evidence *exclusively concern* D_1 (for A_1) and D_2 (for A_2), it is, obviously, still correct to express A_1's and A_2's representors, α_1 and α_2, via the propositional atoms associated with the respective domains, D_1 and D_2, about which they have evidence. As with the original example, P_{1*2} remains a rational response to the aggregate of A_1's and A_2's evidence (or at least $P(\alpha_1 \wedge \alpha_2) \geq r$ is not a rational response). The other features of the example are identical to that of the original example, so that the list of identities enumerated by (i) through (vi) hold. As such, the present variant of the example permits us to reason to a contradiction (in the same manner as the original example), giving us grounds to reject (DC).

6. Conclusion

The proposed argument against (DC) is not decisive, inasmuch as the argument presupposes the truth of the Relevant Factors Principle, and the principle is dubitable. The state of the dialectic is clarified, if we consider a reformulation of the reductio of section 4 (and section 5) where the Relevant Factors Principle is among the assumptions of the reductio. In that case, the reductio yields the conclusion that (at least) one of (DC) and the Relevant Factors Principle is false. The conclusion of the argument thereby yields a reason to reject (DC), whose strength is in proportion to the plausibility of the Relevant Factors Principle.

Considerations bearing on the plausibility of the Relevant Factors Principle were presented in section 3, and are summarized as follows: A truncated version of the Relevant Factors Principle would have been quite plausible if we had only included factors (I) and (II). It is hard to see why we should think that factors apart from evidence, and the manner in which a belief is based upon evidence, could be relevant to determining the epistemic rationality of holding a belief. But I have striven to be ecumenical, and I do not want to beg the question against advocates of (DC), such as Leitgeb, who explicitly maintain the relevance of contextual factors (namely, (III), (IV), (V), (VI), and (VII)) that are obviously independent of (I) and (II). For this reason the Relevant Factors Principle explicitly mentions the possible relevance of factors (I) through (VI), and even (VII). It is also important to note that a failure of the Relevant Factors Principle to explicitly mention some factor does not imply that the factor is not implicitly accommodated by the principle. In general, for any given factor F, the Relevant Factors Principle does not foreclose on the possibility that F is relevant to determinations of the rationality of respective beliefs. The principle simply requires that the relevance of F derives from, or operates through, the factors that the principle explicitly mentions. This explains why advocates of (DC) do not have a prior commitment to reject or deny the plausibility of the Relevant Factors Principle, given that the conflict between (DC) and the principle is not transparent.

Even if the Relevant Factors Principle is false, its inconsistency with (DC) implies that if (DC) is correct, then the factors that underpin its satisfaction are *sui generis* relative to the factors explicitly and implicitly captured by the Relevant Factors Principle (which are inclusive of practical and evidential considerations, and much more). This places a burden on the advocate of (DC) to explain the source of the prescription expressed by (DC), given that the prescription is unmotivated by the factors that have traditionally been viewed as relevant to determinations of rational belief.[15]

In attempting to evade or defuse the argument of the present article, one might deny that there is a single univocal concept of rational belief. And, of course, if there are multiple conceptions of rational belief, then it may be that apparent disputes about the nature of rational belief (e.g., whether rational belief sets are closed under deductive consequences) are not genuine disputes, but merely reflect the explication of distinct concepts. In fact, I am open to the possibility that there are distinct conceptions of rational belief, and even to the possibility that these alternative conceptions have prescriptive force with respect to distinct kinds of belief (cf. Pettigrew, 2015). However, the argument presented in this article proceeds from very modest assumptions concerning the nature of rational belief, namely, the Relevant Factors Principle, and a rather unobjectionable variant of the left-to-right direction of the Lockean Thesis (requiring only that r be greater than zero). Inasmuch as it is plausible to regard these modest assumptions as applying to all sensible conceptions of rational belief, the argument of the present article puts significant pressure upon any explication of a concept of rational belief that maintains that rational belief sets are closed under deductive consequences.

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15 While the defender of (DC) may have something to say on this count, I doubt that a story can be told that is capable of withstanding close scrutiny, given that the story must not appeal to factors that are explicitly or implicitly encoded by the Relevant Factors Principle, such as to considerations of evidence, or to practical or social functions that deductive closure may be thought to enable. Whether any of the considerations that have been adduced in favour of (DC) are able to meet these requirements is unclear (but see Christensen (2004) for a survey).

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